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Volatility Lessons

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Abstract

The average monthly premium of the Market return over the one-month T-Bill return is substantial, as are average premiums of value and small stocks over Market. As the return horizon increases, premium distributions become more disperse, but they move to the right (toward higher values) faster than they become more disperse. There is, however, some bad news. Even if future expected premiums match high past averages, high volatility means that for the three- and five-year periods commonly used to evaluate asset allocations, the probabilities of negative realized premiums are substantial, and the probabilities are nontrivial for ten-year and 20-year periods.

Stock returns are volatile. For July 1963 to December 2016 (henceforth 1963-2016) the equity premium – the difference between the monthly return on the market portfolio of U.S. common stocks (Market) and the one-month U.S. Treasury bill rate averages a hefty 0.51% per month (about 6% per year). But driven by volatility in the Market return, the standard deviation of the monthly equity premium, 4.42%, is almost nine times larger. Even if future monthly returns are from the distribution of observed monthly returns, so the expected premium is positive and large, the high volatility of monthly returns creates substantial uncertainty about whether future realized premiums will be positive even over relatively long periods.

Value stocks and small stocks have higher average returns than Market. The average return on a value-weight (cap-weight) portfolio of NYSE, AMEX, and NASDAQ stocks with book-to

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market equity ratios (*B*/*M*) above the 70th percentile of *B*/*M* for NYSE stocks exceeds the average Market return for 1963-2016 by 0.29% per month. The standard deviation of this Market Value premium is a substantial 2.19% per month. The 1963-2016 average return on the value-weight portfolio of Small stocks (below the median NYSE market cap) is 0.27% per month greater than the average Market return and the standard deviation of the premium is 2.82%. Again, even if all future monthly returns come from the set of realized returns that produced the large positive Market Value or Small premium, the high volatility of monthly premiums implies lots of uncertainty about whether future premiums are positive even over relatively long periods.

The high volatility of stock returns is common knowledge, but many professional investors seem unaware of its implications. Negative equity premiums and negative premiums of value and small stock returns relative to Market are commonplace for three- to five-year periods, and they are far from rare for ten-year periods. Moreover, there is nothing special about the three US equity portfolios we examine. We find similar results, for example, for a value-weight portfolio of developed market stocks outside the US. Our general message is universal; because of the high volatility of stock returns, investors cannot draw strong inferences about expected returns from three, five, or even ten years of realized returns. Those who act on such noisy evidence should reconsider their approach.

We are, of course, not the first to study the distribution of stock returns. The References at the end of the paper list many related papers, theoretical as well as empirical.

1. Volatility and the equity premium

We first examine how the distribution of the U.S equity premium changes as the return horizon increases from one month to thirty years. The monthly equity premium is the monthly Market return minus the return on a one-month U.S. Treasury bill (T-Bill). The data are from the Center for Research in Security Prices (CRSP) of the University of Chicago Booth School of

Business. CRSP data start in 1926, but we focus on the July 1963 to December 2016 (henceforth 1963-2016) period of roughly constant Market return volatility. (See Fama and French 2017 for details.) Market is the CRSP value-weight (cap-weight) portfolio of NYSE, AMEX, and (after 1972) NASDAQ stocks.

We examine distributions of monthly, annual, three-, five-, ten-, 20-, and 30-year equity premiums. The sample of 642 months is large for monthly returns. Sample sizes shrink, however, as the horizon is extended, and it is difficult to identify patterns in distributions of long-horizon returns. We use a common statistical tool – bootstrap simulations – to fill in details of long-horizon return distributions.

We use two approaches to the simulations. The first more standard approach treats the monthly returns of 1963-2016 as the population of returns. To produce the simulated distribution of annual equity premiums, for example, we draw 100,000 paired trials of 12 monthly Market and T-Bill returns. The pairs of monthly Market and T-Bill returns are drawn randomly with replacement from the monthly returns of 1963-2016. Since every month of the sample period has the same chance of being the next one drawn, monthly returns in a trial are independent and identically distributed. We compound each trial's 12 monthly Market and 12 monthly T-Bill returns to get annual returns. The annual equity premium for a trial is the difference between its annual Market and T-Bill returns. The result of this process is 100,000 annual premiums from a sample of 642 pairs of monthly returns.

We produce bootstrap samples of three-, five-, ten-, 20-, and 30-year equity premiums in the same way. Our evidence on the five-year equity premium, for example, is from 100,000 simulated five-year premiums. To construct each premium, we draw 60 months randomly with replacement from the months of 1963-2016, compound the 60 Market returns and 60 T-Bill returns for those months, and then compute the difference between the simulated five-year returns.

Treating the 1963-2016 sample of monthly returns as the population of monthly returns assumes average returns for 1963-2016 are expected returns (the true means of the populations of returns). Our second simulation approach acknowledges that the average monthly equity premium is only an estimate of the true expected premium and builds uncertainty about the expected premium into the simulated premiums. Specifically, for each of a horizon's 100,000 trials, we simulate errors in our estimates of the expected monthly Market and T-Bill returns with two correlated, mean-zero normal random variables, X_1 and X_2. Since X_1 and X_2 represent our uncertainty about the expected monthly Market and T-Bill returns, their standard deviations are the standard errors of the mean returns for 1963-2016, 0.17% and 0.01%. To compute a trial's long-horizon adjusted premium, we add X_1 and X_2 to the monthly market and T-Bill returns drawn in the trial, compound the adjusted returns, and take the difference. (We construct X_1 and X_2 so their correlation matches the correlation between monthly Market and T-Bill returns for 1963-2016. As a result, the cross-trial standard deviation of the expected monthly equity premium matches the standard error of the mean of the monthly equity premiums for 1963-2016.)

As noted above, the 1963-2016 average value of the monthly equity premium is large, 0.51%, but the standard error of the mean (a measure of uncertainty about the underlying true mean) is 0.17%. The mean is thus 2.92 standard errors from zero, so we are fairly certain the true mean is positive. It could be rather far, however, above or below 0.51%. If we assume the expected equity premium is constant (though unknown), the standard deviation of the unexpected monthly premium must be the standard deviation of the realized monthly premium, 4.42%. This is much larger than the standard error of the mean, 0.17%, our estimated uncertainty about the expected premium. We see shortly that for shorter return horizons (five years or less), uncertainty about the equity premium is dominated by its unexpected component; uncertainty about the expected premium contributes little to return dispersion. As Pastor and Stambaugh (2012) argue, however, uncertainty about the expected premium is more consequential for longer return horizons.

Table 1 summarizes distributions of equity premiums constructed from 1963-2016 returns. For each return horizon, the table shows average premium (*Mean*), standard deviation of premiums (*SD*), measures of skew (*Skew*) and kurtosis (*Kurt*), the percent of negative premiums (% Neg), and percentiles (1^{st}, 5^{th}, 25^{th}, 75^{th}, 95^{th}, 99^{th}) of the distribution of premiums. The Monthly row summarizes the distribution of actual monthly premiums. The other rows summarize the distributions of 100,000 simulated premiums for longer horizons. Panel A of the table shows results with a known expected monthly equity premium equal to the sample average for 1963-2016. The results in Panel B add uncertainty about the expected premium. We focus initially on the results in Panel A.

If a distribution is symmetric about its mean, *Skew* (the third moment about the mean, divided by the cubed standard deviation) is zero. *Skew* is -0.53 for monthly equity premiums, so the distribution of monthly premiums is skewed to the left. *Skew* is positive, 0.25, for annual premiums and increases a lot for longer return horizons, to 3.78 for 30-year premiums.

Monthly equity premiums are leptokurtic, which means there are more extreme returns than we would expect from a normal distribution. *Kurt* (the fourth moment about the mean, divided by the standard deviation to the fourth power) is 3.0 for a normal distribution and it is 4.97 for monthly equity premiums of 1963-2016. *Kurt* falls to 3.19 for annual premiums, but rises strongly for longer return horizons, to 32.37 for 30-year returns.

The standard deviation of equity premiums increases with the return horizon, from 17% for annual premiums to 2551% for 30-year premiums. In general, the distribution of premiums moves to the right faster than its dispersion increases. For example, the median increases 231 fold, from 6% for annual premiums to 1390% for 30-year premiums. Together, increasing kurtosis and right skew for longer horizons spread out the good outcomes in the right tail more than the bad outcomes in the left. For example, the fifth and 95th percentiles of monthly premiums for 1963-2016 are near equidistant from the median, but the 95th percentile of simulated 20-year premiums is 1424 percentage points from the median, versus 471 percentage points for the fifth percentile. Despite the general shift to the right, increasing dispersion pushes the extremely bad outcomes further to the left for longer horizons. The 1st percentile falls monotonically from -12% at one month to -161% at 30 years. (Premiums less than -100% are possible because they are differences between two returns.)

Most results described above are apparent in Figures 1a to 1f, which plot distributions of 1-, 3-, 5-, 10-, 20-, and 30-year equity premiums from the bootstrap simulations. For perspective, the figures also show the matched normal distribution with the same mean and standard deviation as the equity premiums for a horizon.

Confirming *Skew* (0.25) and *Kurt* (3.19), the distribution of annual equity premiums is closest to its matched normal, but it has some right skew and excess kurtosis relative to the normal. Excess kurtosis shows up as excess observations around the peak of the distribution and in the extreme tails. Right skew shows up as a peak for the distribution of annual equity premiums that is slightly to the left of the mean (the peak of the normal curve), and an extreme right tail with more observations (relative to the matched normal) than the extreme left tail.

As the return horizon increases and right skew and kurtosis become more severe, these characteristics of equity premium distributions become more accentuated and thus more obvious in Figure 1. The distribution of equity premiums drifts to the right and becomes more disperse (note the changes in the horizontal scale). Equity premium distributions become more peaked relative to their matched normals, and the peaks move further to the left of the means of the matched normals. Relative to the matched normals, there are too few observations in the near right tails of long-horizon premium distributions, but there are too many in the extreme right tails (really good outcomes). As the return horizon increases, extreme left tails (bad outcomes) hollow out. Thus, bad outcomes generally become less likely and extreme good outcomes become more likely for longer return horizons. The percent of negative equity premiums falls from 41.28% for the monthly returns of 1963-2016 to 4.08% for 30-year returns (Table 1).

Most of the news about equity premium distributions for longer return horizons in Panel A of Table 1 is good, but there is bad news. For the three- and five-year periods that are often the focus of professional investors, negative equity premiums occur in 28.54% (three years) and 23.41% (five years) of simulation runs. Even for ten- and 20-year periods, negative premiums occur in 15.60% and 7.88% of simulation runs. These probability estimates are from simulations that use the realized monthly returns of 1963-2016 to construct long-horizon returns. Each month's expected return in the simulations is the large positive average return of 1963-2016, 0.51%, so we know any negative equity premiums are chance results, irrelevant for predictions about future equity premiums.

Investors, of course, do not know the true expected equity premium. Panel B of Table 1 adds uncertainty about the expected premium to the results in Panel A. Median outcomes are largely unchanged for all horizons, but the added source of return uncertainty increases the dispersion of premiums. For example, the standard deviation of ten-year equity premiums rises from 149% to 167%. The increase in dispersion is not symmetric: the right tail moves out more than the left. For example, adding uncertainty about the expected premium increases the 95th percentile of 20-year equity premiums from 1857% to 2286% (an extra return of 429% on an investment in Market financed by T-Bills), but the 5th percentile falls from -38% to -77% (an extra loss of 39%). With uncertainty about the expected premium, *Skew* is systematically higher, and (for horizons beyond a year) *Kurt* is also higher. Thus, outliers are more numerous but primarily in the right tail.

For the return horizons we examine, the effects of uncertainty about the expected equity premium are tiny for horizons of five years or less, but they are progressively larger for horizons of ten years or more. This happens because, under the assumption that the expected equity premium is constant, a mistake in the expected premium repeats every month. Thus, although volatility due to the unexpected component of monthly equity premiums grows roughly like $T^{1/2}$ (T is the number of months in the investment horizon), volatility due to uncertainty about the expected premium grows roughly like T. At short horizons, the much larger volatility of unexpected monthly equity premiums (standard deviation = 4.42%) dominates uncertainty about the expected premium (standard error = 0.17%), but at longer horizons (larger values of T) the cumulative effect of uncertainty about the expected premium becomes relatively more important.

For horizons of five years or less, uncertainty about the expected equity premium has little effect on frequencies of negative premiums. For horizons of ten years or more the increases in frequencies of negative premiums are more substantial. For example, adding uncertainty about the expected premium increases the estimated probability of negative 20-year premiums from 7.88% to 11.22%; the estimate for 30-year premiums nearly doubles, from 4.08% to 8.02%. The changes are due to simulation runs in which the estimate of the expected monthly equity premium, though almost surely positive, is less than the sample average, 0.51%.

2. Value and small stock premiums relative to Market

The Market portfolio is an efficient portfolio relevant for choice by investors in all common asset pricing models. Investors are often encouraged to tilt away from Market toward, for example, value stocks or small stocks, which in past data offer premiums relative to Market. We next examine bootstrap distributions of premiums relative to Market for three value portfolios (Market

Value, Big Value, and Small Value) and a Small stock portfolio with no value tilt. Small stocks are NYSE, AMEX, and NASDAQ stocks below the median market cap for NYSE stocks, and big stocks are above the NYSE median. Value portfolios include NYSE, AMEX, and NASDAQ stocks above the 70th percentile of the book-to-market equity ratio (B/M) for NYSE stocks. Small Value stocks are both small and value, Big Value stocks are big and value, and Market Value includes all value stocks. The portfolios are value-weight and are rebalanced at the end of June each year. Size is market cap at the end of June, and B/M is book equity for the fiscal yearend in the previous calendar year divided by market cap at the end of December of the previous year.

Table 2 reports 1963-2016 average monthly returns in excess of Market for the four portfolios. They range from 0.20% (t = 2.13) for Big Value to 0.52% (t = 4.35) for Small Value (Table 2). Market Value's premium, 0.29% (t = 3.35), is between those for Small Value and Big Value, and Small beats Market by 0.27% (t = 2.40). The t-statistic for Market's 1963-2016 average monthly equity premium of 0.51% is 2.92, so by that criterion we should be at least as confident that the expected premiums on Market Value and Small Value are positive as we are that the expected equity premium is positive. The t-statistics for Big Value and Small are smaller, but the expected premiums for these portfolios are still likely positive.

The simulations for long horizon value and Small premiums in excess of Market are like those for the equity premium, except we draw paired random samples of portfolio and Market returns. To allow the expected premium to vary randomly across a horizon's 100,000 simulated premiums, we add two correlated, mean-zero normal random variables to the monthly portfolio and Market returns that are compounded to produce a premium. The standard deviations of the two random expected return adjustments match the standard errors of the means of the full sample of monthly portfolio and Market returns, and the correlation between them matches the correlation between the observed portfolio and Market returns. As a result, the standard deviation of the difference between each trial’s expected return adjustments is the standard error of the mean difference between monthly portfolio and Market returns for 1963-2016. We add a trial’s expected return adjustments to the portfolio and Market returns for the months used in the trial, cumulate the two sets of adjusted returns, then compute the simulated premium. Panel B of Table 3 summarizes the results. Panel A summarizes simulations in which there is no adjustment for uncertainty about expected premiums.

The properties of value and Small returns in excess of Market for longer horizons, in Table 3, are like those of the equity premium, so the discussion can be brief. The distributions of the premiums become more disperse and right skewed for longer return horizons and kurtosis increases. Except for the extreme left tail, premium distributions move to the right faster than they become more disperse. Increasing right skew means more of the dispersion is toward good outcomes, and the combination of increasing right skew and kurtosis means outliers are primarily in the right tail. Allowing for uncertainty about expected premiums increases the dispersion of premiums, but more right skew and kurtosis again mean the right tail spreads out more than the left.

Perhaps the most interesting results in Table 3 are the frequencies of negative premiums over Market. When evaluating portfolio performance, professional investors often put much weight on three or five years of past returns. The message from Table 3 is that chance is a big player in outcomes for these horizons. In the simulations of Table 3 Panel A, expected monthly premiums are equal to the large positive sample averages in Table 2. Negative simulated premiums are then just chance results of high return volatility. For a five-year investment horizon, the estimated probabilities that value and Small premiums over Market are negative on a purely chance basis are substantial, ranging from 11.68% for Small Value (the big winner in Table 3) to 29.80% for Small. The estimated frequencies of negative premiums are bigger at three years, between

18.00% and 34.06%, even though we know that by construction the expected premiums are positive and large. Table 3 says negative premiums relative to Market are less likely but still substantial for ten-year periods (ranging from 4.53% for Small Value to 22.48% for Small). Small Value aside, they are also nontrivial for 20-year periods (from 0.82% for Small Value to 14.41% for Small).

The Panel B simulations allow for uncertainty about expected premiums, but the effects of this adjustment on distributions of premiums and frequencies of negative premiums are again minor for return horizons of five years or less. For longer horizons, uncertainty about expected premiums has a bigger effect on the frequency of negative outcomes. For example, the frequency of negative 20-year premiums of Small over Market rises from 14.41% to 18.33%. This is the effect of the possibility that, though most likely positive, the expected premium might be smaller than the sample average.

The good news is that increasing right skew of premium distributions as the return horizon is extended means good outcomes in the right tails of premium distributions become more extreme relative to bad outcomes in the left tail. For example, at the five-year horizon, $1 invested in Market Value loses to $1 invested in Market by $0.20 or more in 5% of simulation runs (Table 3 Panel B), but Market Value wins by $1.05 or more in 5% of simulation runs. At the ten-year horizon, the 5th percentile of the premium distribution falls from -$0.20 to -$0.26 (the bad outcome is slightly worse), but the 95th percentile rises from $1.05 to $3.86 (the good outcome is much better). Big Value produces the smallest premiums over Market, and at the five-year horizon $1 invested in Big Values loses by $0.33 or more to $1 invested in Market in 5% of simulation runs, but Big Value wins by $0.90 or more in 5% of simulation runs. At the ten-year horizon, the 5th percentile of the premium distribution for Big Value is a bit worse, dropping from -$0.33 to -$0.65, but the 95th percentile is much better, rising from $0.90 to $3.08.

In short, value and small stock premiums over Market are always risky, but for longer return horizons, good outcomes become more likely and more extreme relative to bad outcomes.

Conclusions

The average monthly premium of the Market return over the one-month T-Bill return is substantial, as are average monthly premiums of value stocks and small stocks over Market. As the return horizon is extended, premium distributions move to the right (toward higher values) and become more disperse, but in general they move to the right faster than they become more disperse. Also good news, right skew and kurtosis increase for longer return horizons. This means that relative to matched normal distributions, distributions of premiums become more peaked around their medians, their left tails hollow out, and they have an excess of outcomes in their extreme right tails.

Most of the evidence about the evolution of premium distributions for longer return horizons is good, but there is some bad news. The high volatility of monthly stock returns and premiums means that for the three-year and five-year periods used by many professional investors to evaluate asset allocations, the probabilities that premiums are negative on a purely chance basis are substantial, and they are nontrivial even for ten-year and 20-year periods.

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Table 1 – Simulations of Equity Premium for July 1963 to December 2016, 642 months

The Monthly equity premium is the difference between the monthly return on the value-weight portfolio of NYSE-AMEX-NASDAQ stocks (Market) and the 1-month Treasury bill rate. The results for 1 Year to 30 Years summarize 100,000 bootstrapped premiums. *Ave* is the average, *SD* is the standard deviation, *Skew* is the skewness, and *Kurt* is the kurtosis of actual or bootstrapped premiums for each horizon. *Neg* is the percent of premiums that are negative. The table also shows percentiles of the premiums for each horizon. The simulations in Panel B allow for uncertainty about expected premiums. Averages, standard deviations, and percentiles are dollar payoffs (not percent returns) from borrowing one dollar at the Treasury bill rate and investing it in Market.

						Percentiles						
	Ave	*SD*	*Skew*	*Kurt*	*Neg*	1%	5%	25%	50%	75%	95%	99%
Panel A: Simulations with known expected premium												
Monthly	0.01	0.04	-0.53	4.97	41.28	-0.12	-0.07	-0.02	0.01	0.03	0.07	0.11
1 Year	0.07	0.17	0.25	3.19	35.97	-0.30	-0.20	-0.05	0.06	0.18	0.36	0.50
3 Years	0.23	0.37	0.70	3.93	28.54	-0.45	-0.30	-0.03	0.19	0.45	0.90	1.28
5 Years	0.45	0.59	0.98	4.75	23.41	-0.56	-0.35	0.02	0.36	0.78	1.54	2.23
10 Years	1.33	1.49	1.57	7.61	15.60	-0.79	-0.44	0.29	1.03	2.03	4.14	6.34
20 Years	6.10	6.58	2.57	15.89	7.88	-1.23	-0.38	1.77	4.33	8.38	18.57	30.55
30 Years	21.39	25.51	3.78	32.37	4.08	-1.61	0.37	6.12	13.90	27.50	66.73	122.79
Panel B: Simulations with uncertain expected premium												
1 Year	0.07	0.17	0.26	3.19	36.21	-0.30	-0.20	-0.05	0.06	0.18	0.36	0.50
3 Years	0.23	0.38	0.73	3.99	29.12	-0.47	-0.31	-0.04	0.19	0.46	0.92	1.32
5 Years	0.46	0.63	1.03	4.89	24.51	-0.59	-0.38	0.01	0.36	0.80	1.62	2.37
10 Years	1.40	1.67	1.73	8.51	17.79	-0.87	-0.52	0.23	1.03	2.14	4.55	7.16
20 Years	6.85	8.70	3.20	22.68	11.22	-1.54	-0.77	1.42	4.31	9.24	22.86	40.39
30 Years	26.69	42.39	5.69	67.62	8.02	-2.60	-0.93	4.76	13.86	32.03	95.47	196.29

Table 2 – Summary statistics for monthly value and small stock returns in excess of Market for July 1963 to December 2016, 642 months

Small stocks are NYSE, AMEX, and NADSAQ stocks below the median NYSE market cap, and big stocks are above the NYSE median. Value portfolios include NYSE, AMEX, and NASDAQ stocks above the 70th percentile of the book-to-market equity ratio (B/M) for NYSE stocks. Small Value stocks are both small and value, Big Value stocks are big and value, and Market Value includes all value stocks. Small is the portfolio of all small stocks with no value tilt. The portfolios are value-weight and are reconstructed at the end of June each year. Size (Small or Big) is market cap at the end of June, and *B*/*M* is book equity for the fiscal yearend in the previous calendar year divided by market cap at the end of December of the previous year. The table shows average return (Mean), the standard deviation of returns (Std Dev), the standard error of the mean (SE Mean), and *t*-Mean = Mean/SE Mean. Means, standard deviations, and standard errors are in percent.

	Market Value	Big Value	Small Value	Small
Mean	0.29	0.20	0.52	0.27
Std Dev	2.19	2.32	3.02	2.82
SE Mean	0.09	0.09	0.12	0.11
t-Mean	3.35	2.13	4.35	2.40

Table 3 – Summary statistics for simulations of simple dollar premiums in excess of Market for Market Value, Big Value, Small Value, and Small: July 1963 to December 2016, 642 months

Small stocks are NYSE, AMEX, and NADSAQ stocks below the median market cap for NYSE stocks, and big stocks are above the NYSE median. Value portfolios include NYSE, AMEX, and NASDAQ stocks above the 70th percentile of the book-to-market equity ratio (B/M) for NYSE stocks. Small Value stocks are both small and value, Big Value stocks are big and value, and Market Value includes all value stocks. Small is the portfolio of all small stocks with no value tilt. The portfolios are value-weight (VW) and are reconstructed at the end of June each year. For each return horizon, the table shows average premium (*Ave*), standard deviation of premiums (*Std*), skewness (*Skew*) and kurtosis (*Kurt*), the percent of negative premiums (*Neg*), and percentiles of the distribution of premiums. The Monthly row in each block summarizes the distribution of actual monthly returns. Each of the remaining rows summarizes the distribution of returns produced by 100,000 simulations of returns for a particular horizon. The simulations in Panel B allow for uncertainty about expected premiums.

Panel A: Simulations with known expected premium

						Percentiles						
	Ave	*SD*	*Skew*	*Kurt*	*Neg*	1%	5%	25%	50%	75%	95%	99%
Premiums of Market Value over Market												
Monthly	0.00	0.02	0.39	6.37	46.26	-0.06	-0.03	-0.01	0.00	0.01	0.04	0.07
1 Year	0.04	0.09	0.36	3.76	33.13	-0.15	-0.10	-0.02	0.03	0.09	0.19	0.27
3 Years	0.15	0.21	0.70	4.66	23.23	-0.27	-0.15	0.01	0.13	0.27	0.51	0.75
5 Years	0.32	0.38	1.08	6.05	17.10	-0.38	-0.18	0.07	0.27	0.51	1.00	1.49
10 Years	1.20	1.23	1.95	10.65	8.95	-0.64	-0.18	0.40	0.93	1.70	3.51	5.54
20 Years	8.52	9.54	3.67	31.68	2.85	-0.91	0.47	2.76	5.79	10.96	25.65	45.24
30 Years	45.75	61.14	5.40	66.56	0.96	0.09	3.34	12.86	27.34	55.53	147.75	291.87
Premiums of Big Value over Market												
Monthly	0.00	0.02	0.05	6.47	47.66	-0.06	-0.03	-0.01	0.00	0.01	0.04	0.07
1 Year	0.03	0.09	0.23	3.58	39.22	-0.18	-0.12	-0.03	0.02	0.08	0.18	0.26
3 Years	0.10	0.21	0.51	4.39	32.59	-0.36	-0.22	-0.04	0.08	0.22	0.47	0.69
5 Years	0.21	0.37	0.84	5.56	27.78	-0.54	-0.30	-0.02	0.17	0.41	0.86	1.31
10 Years	0.77	1.10	1.80	11.26	20.49	-1.17	-0.55	0.09	0.56	1.22	2.76	4.54
20 Years	5.12	7.11	3.52	28.95	12.24	-3.80	-1.13	1.05	3.20	6.96	17.74	32.85
30 Years	25.67	39.37	5.50	69.80	7.83	-9.60	-1.57	5.19	14.13	31.89	90.37	180.91
Premiums of Small Value over Market												
Monthly	0.01	0.03	0.52	5.55	42.68	-0.07	-0.04	-0.01	0.00	0.02	0.05	0.10
1 Year	0.07	0.13	0.61	4.23	29.59	-0.18	-0.11	-0.01	0.06	0.14	0.29	0.42
3 Years	0.28	0.33	1.09	5.85	18.00	-0.30	-0.15	0.06	0.23	0.45	0.88	1.29
5 Years	0.61	0.63	1.56	8.12	11.68	-0.37	-0.15	0.18	0.49	0.90	1.79	2.72
10 Years	2.49	2.51	2.63	16.90	4.53	-0.46	0.03	0.86	1.83	3.34	7.17	11.74
20 Years	20.96	25.44	4.14	35.11	0.82	0.15	1.70	6.30	13.17	26.17	65.30	122.37
30 Years	135.47	206.41	6.66	106.37	0.17	3.03	9.25	32.82	72.59	157.79	458.66	967.29
Premiums of Small over Market												
Monthly	0.00	0.03	0.43	6.30	47.20	-0.07	-0.04	-0.01	0.00	0.02	0.05	0.08
1 Year	0.04	0.11	0.66	4.26	40.53	-0.19	-0.13	-0.04	0.03	0.10	0.24	0.36
3 Years	0.14	0.28	1.18	6.02	34.06	-0.35	-0.23	-0.05	0.09	0.28	0.65	1.01
5 Years	0.29	0.51	1.65	8.28	29.80	-0.50	-0.30	-0.04	0.19	0.52	1.24	2.03
10 Years	1.11	1.72	3.02	23.15	22.48	-1.00	-0.52	0.05	0.64	1.63	4.27	7.56
20 Years	7.64	13.15	6.73	158.08	14.41	-2.84	-1.08	0.89	3.67	9.46	29.22	60.67
30 Years	40.24	79.73	8.25	154.98	9.62	-7.07	-1.69	4.68	16.40	44.43	157.81	354.75

Table 3 (continued)

Panel B: Simulations with uncertain expected premium

	Ave	*SD*	*Skew*	*Kurt*	*Neg*	Percentiles 1%	5%	25%	50%	75%	95%	99%
Premiums of Market Value over Market												
1 Year	0.04	0.09	0.36	3.73	33.19	-0.15	-0.10	-0.02	0.03	0.09	0.19	0.27
3 Years	0.15	0.21	0.70	4.67	23.95	-0.29	-0.16	0.01	0.13	0.27	0.53	0.77
5 Years	0.32	0.40	1.14	6.25	18.29	-0.42	-0.20	0.06	0.27	0.52	1.05	1.57
10 Years	1.25	1.41	2.25	13.62	10.90	-0.81	-0.26	0.35	0.92	1.77	3.86	6.36
20 Years	9.62	13.40	4.87	56.05	5.22	-2.16	-0.04	2.31	5.61	11.93	32.38	63.65
30 Years	59.14	116.78	9.74	193.11	3.11	-4.60	1.15	9.95	26.36	64.37	216.02	502.21
Premiums of Big Value over Market												
1 Year	0.03	0.09	0.23	3.59	39.32	-0.19	-0.12	-0.03	0.02	0.08	0.18	0.27
3 Years	0.10	0.22	0.53	4.46	33.05	-0.37	-0.23	-0.04	0.08	0.23	0.48	0.71
5 Years	0.22	0.39	0.90	5.80	28.66	-0.57	-0.33	-0.03	0.17	0.42	0.90	1.39
10 Years	0.80	1.26	2.07	13.94	22.44	-1.41	-0.65	0.05	0.55	1.27	3.08	5.19
20 Years	5.84	10.05	4.80	54.23	16.03	-5.98	-1.93	0.72	3.08	7.63	22.53	46.52
30 Years	33.67	75.79	12.62	454.85	12.64	-21.37	-4.85	3.32	13.07	36.86	136.11	319.71
Premiums of Small Value over Market												
1 Year	0.07	0.13	0.61	4.21	29.82	-0.19	-0.12	-0.02	0.06	0.15	0.29	0.43
3 Years	0.28	0.34	1.10	5.71	18.56	-0.31	-0.16	0.05	0.23	0.46	0.91	1.33
5 Years	0.62	0.67	1.63	8.28	12.77	-0.42	-0.17	0.17	0.49	0.93	1.88	2.93
10 Years	2.61	2.90	3.00	22.32	6.09	-0.63	-0.07	0.78	1.81	3.50	7.92	13.48
20 Years	24.47	38.14	6.91	120.48	2.13	-0.78	0.89	5.42	12.94	28.77	84.82	176.29
30 Years	187.29	422.60	11.29	273.48	0.97	0.06	4.72	25.94	71.02	187.95	711.23	1794.51
Premiums of Small over Market												
1 Year	0.04	0.12	0.66	4.23	40.59	-0.19	-0.13	-0.04	0.02	0.10	0.24	0.36
3 Years	0.14	0.29	1.22	6.30	34.35	-0.36	-0.23	-0.06	0.09	0.28	0.67	1.05
5 Years	0.30	0.54	1.75	8.97	30.51	-0.54	-0.33	-0.05	0.19	0.53	1.32	2.18
10 Years	1.20	2.01	3.57	34.39	24.37	-1.15	-0.61	0.01	0.63	1.73	4.83	8.91
20 Years	9.54	20.24	7.80	138.30	18.33	-4.18	-1.61	0.55	3.59	10.70	39.37	90.79
30 Years	61.71	191.89	20.58	1068.59	14.97	-13.74	-3.92	2.69	15.56	54.30	255.77	706.08

Figure 1 – Histograms of Simulated Equity Premiums with Fitted Normal Distributions, 1963-2016

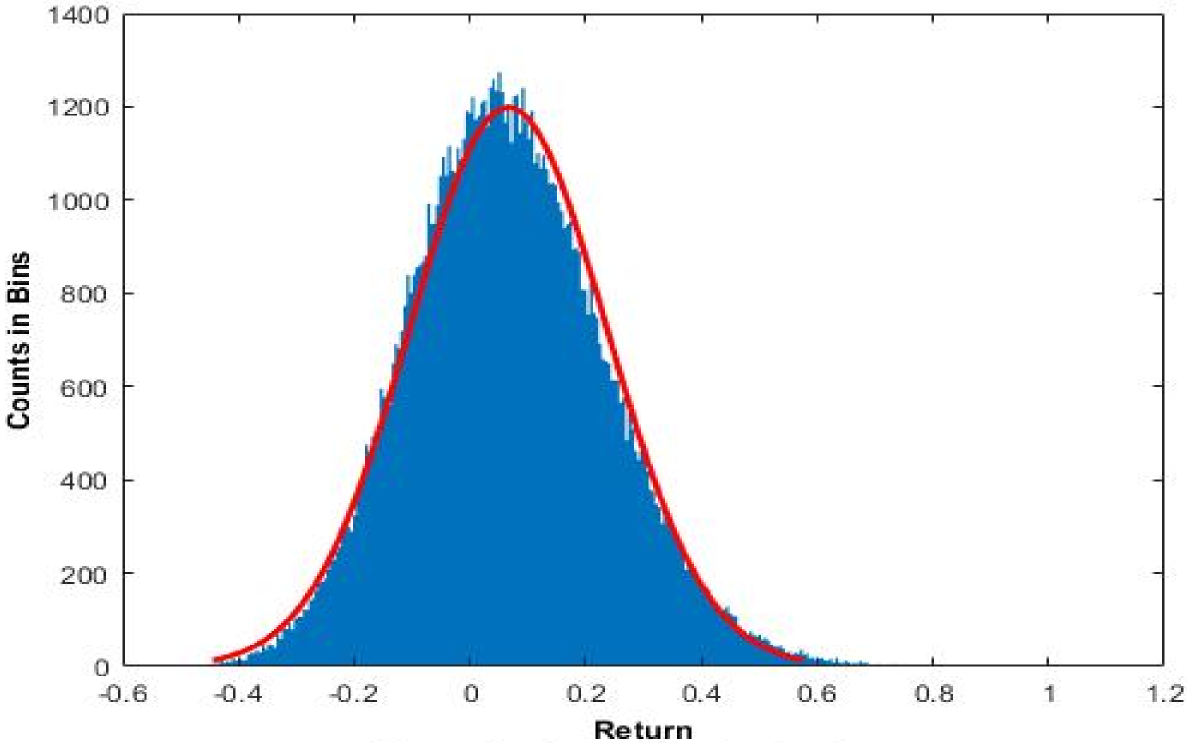

Figure 1a - Annual Equity Premiums

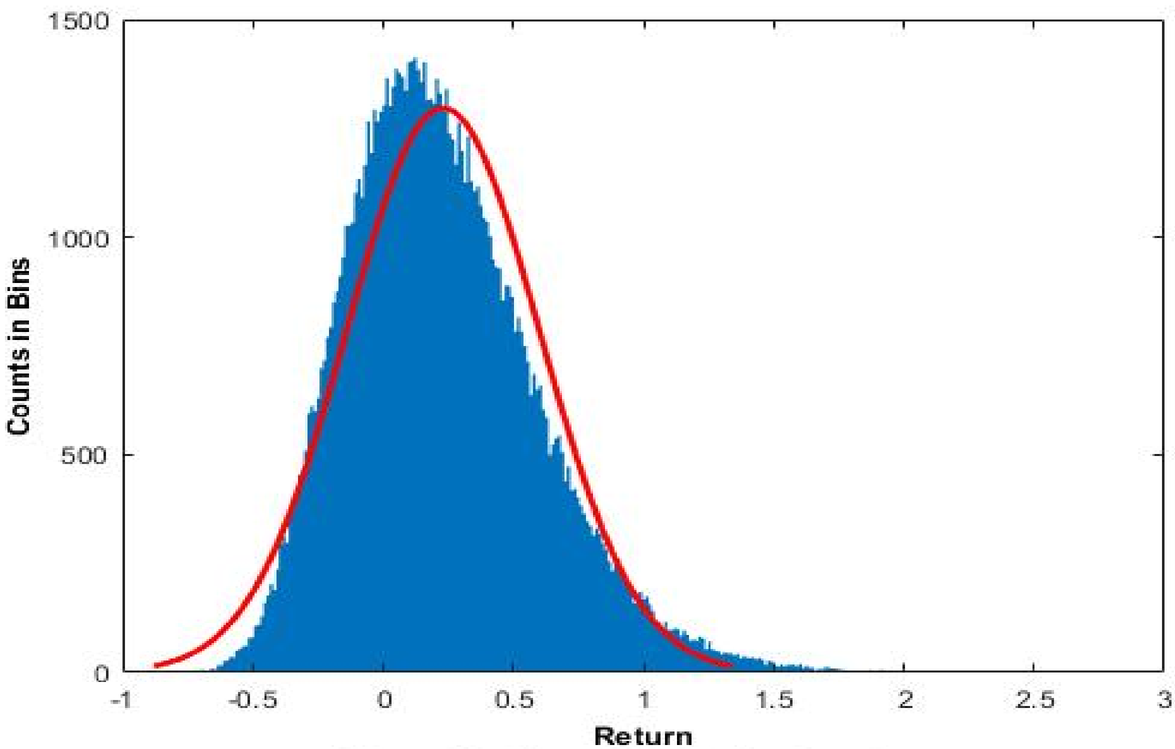

Figure 1b - Three-Year Equity Premiums

Figure 1 (continued)

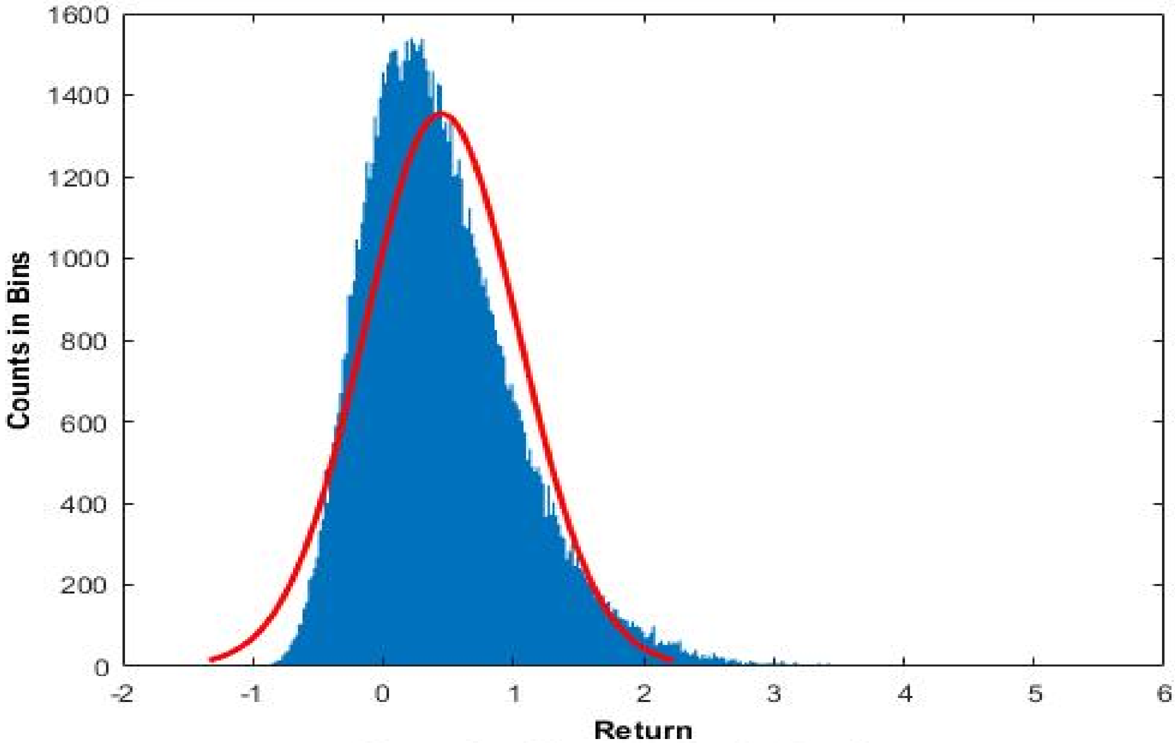

Figure 1c - Five-Year Equity Premiums

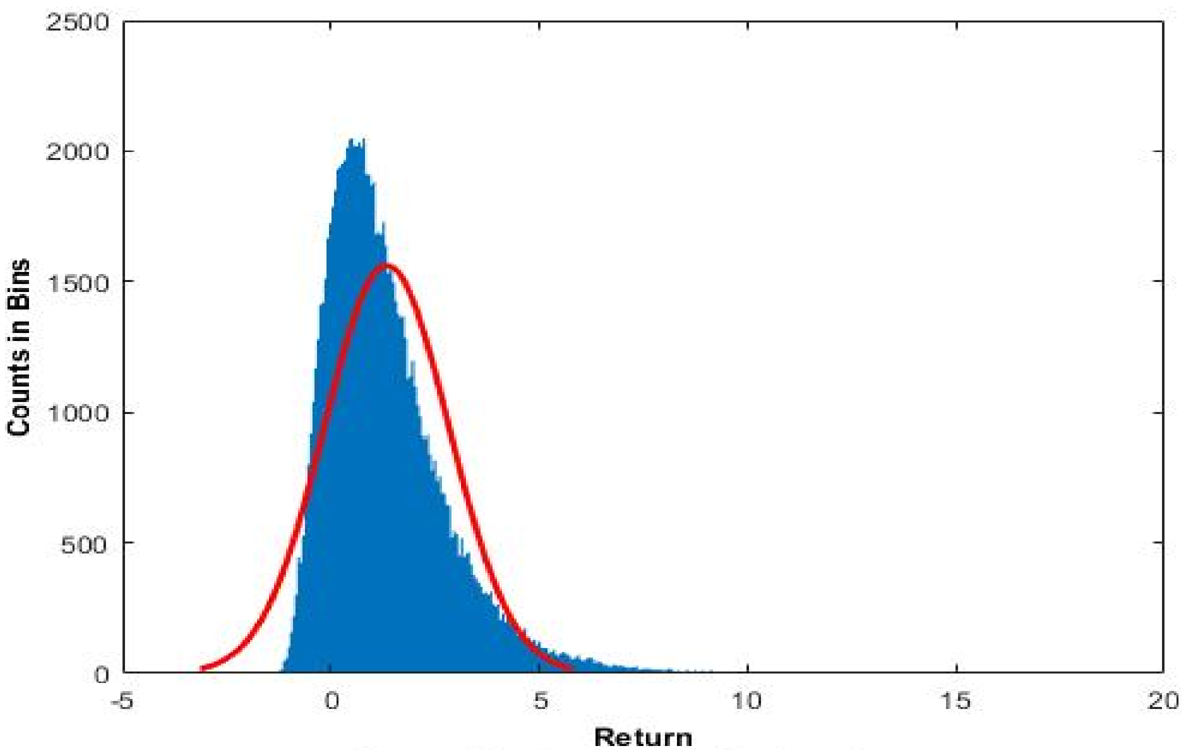

Figure 1d - Ten-Year Equity Premiums

Figure 1 (continued)

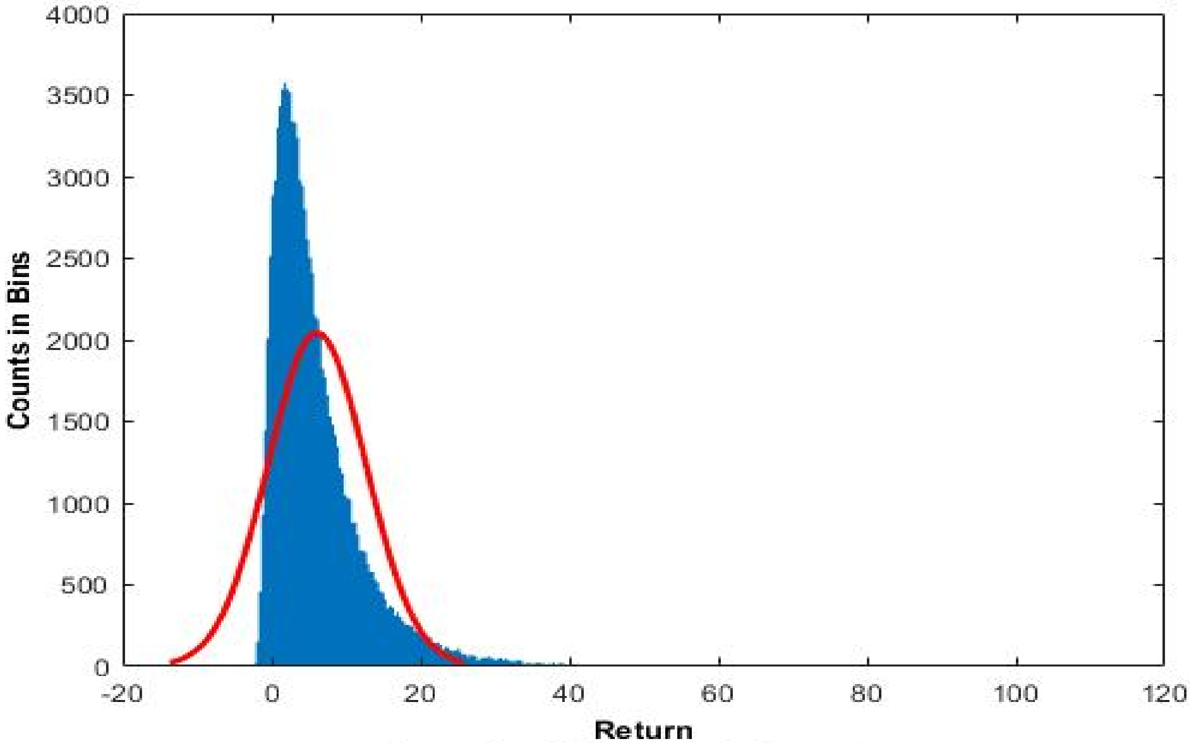

Figure 1e - 20-Year Equity Premiums

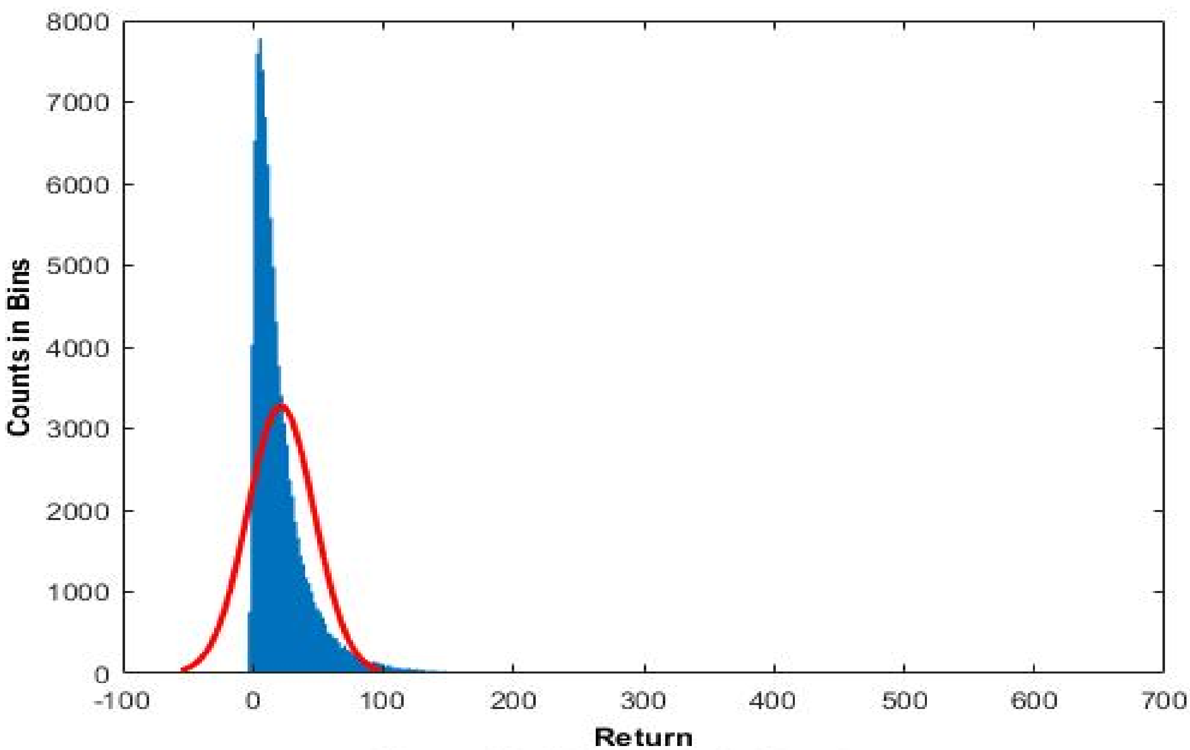

Figure 1f - 30-Year Equity Premiums