

Volatility and Premiums in US Equity Returns

Eugene F. Fama and Kenneth R. French *

Understanding volatility is crucial for informed investment decisions. This paper explores the volatility of the market, size, and value premiums of the Fama-French three-factor model for US equity returns.

Table 1 summarizes the annual premiums. The data are from the Center for Research in Security Prices (CRSP) of the Booth School of Business at the University of Chicago. The CRSP data on NYSE stocks start in 1926; Amex and NASDAQ stocks are added in 1963 and 1973, respectively. The premiums in Table 1 are (i) $RM - RF$, the difference between the market return and the riskfree return, called the equity premium; (ii) SMB (small minus big), the size premium; and (iii) HML (high minus low book-to-market equity), the value premium.

Table 1 also summarizes the annual returns for the components of the premiums. The market return, labeled RM , is the cap-weighted return for all US equities on CRSP, and the riskfree return, RF , is the annual return on a one-month Treasury bill rolled over every month during the year. We use six portfolios to construct SMB and HML. The labels for the six portfolios are descriptive. The first letter, S or B, indicates that the stocks in a portfolio are small (NYSE, Amex, and NASDAQ stocks with market cap below the NYSE median) or big (market cap above the NYSE median). The second letter is G for growth, N for neutral, or V for value. We define growth stocks as NYSE, Amex, and NASDAQ stocks with book-to-market (B/M) equity ratios in the bottom 30% of B/M for NYSE stocks. Neutral stocks are those with B/M in the middle 40% of B/M for NYSE stocks. And value stocks are those with B/M in the top 30% of B/M for NYSE stocks. Flipping the ratio, high B/M is low M/B, so value stocks have low prices relative to book value and growth stocks have high price-to-book ratios.

*Booth School of Business, University of Chicago (Fama) and Amos Tuck School of Business, Dartmouth College (French). Fama and French are also stockholders in, board members of, and consultants to Dimensional Fund Advisors.

We weight the stocks in the six portfolios used to construct SMB and HML by their market caps, but SMB and HML are equal-weight averages of the returns on the six portfolios. Specifically, the size premium, SMB, is the simple average of the returns on the three small-stock portfolios minus the simple average of the returns on the three big-stock portfolios,

$$\text{SMB} = (\text{SG} + \text{SN} + \text{SV})/3 - (\text{BG} + \text{BN} + \text{BV})/3.$$

Likewise, the value premium, HML, is the simple average of the returns on the two value portfolios minus the simple average of the returns on the two growth portfolios,

$$\text{HML} = (\text{SV} + \text{BV})/2 - (\text{SG} + \text{BG})/2.$$

The motivation for equal-weighting the components of SMB and HML is to have a size premium that is neutral with respect to value versus growth (because it equal-weights its growth, neutral, and value components) and a value premium that is neutral with respect to size (because it equal-weights its small and big components).

Our message centers on the importance of the volatility of $\text{RM} - \text{RF}$, SMB, and HML in investment outcomes. To set the stage, Figures 1 to 3 show annual values of the three premiums for the 1927–2011 sample period. Note the scale of the vertical (Return) axis of the equity premium plot in Figure 1, from -60% to 60%. This wide spread is necessary to accommodate year-by-year observations of the annual equity premium. The most extreme returns are in the 1930s, but even if we start in 1950, the annual returns range from -39.99% (2008) to 49.31% (1954). In short, returns to stock investing are volatile. Figures 2 and 3, which plot the equity premium along with the size premium (Figure 2) and the value premium (Figure 3), show that SMB and HML are slightly less volatile than $\text{RM} - \text{RF}$, but the volatility of the size and value premiums is nevertheless high.

Next we need a bit of terminology. When statisticians talk about the true mean of a probability distribution, they call it the expected value. Sample means or averages like those in Table 1 are estimates of expected values. Henceforth, we use the term “expected value” when referring to true means.

For each portfolio, Table 1 shows two statistics. The first is the average annual return (simple, not compounded), labeled Mean. The second is the standard deviation of a portfolio's annual returns, labeled Std, which is a summary measure of the volatility of year-by-year returns. To focus the story, Table 1 shows results only for 1963–2011, which includes NYSE, Amex, and (beginning in 1973) NASDAQ stocks. This is a long 49-year investment lifetime. We stress the concept of an investment lifetime to sharpen the main message. In a nutshell, the message (developed in detail below) is that long time periods are required to be reasonably sure that the average equity premium, the average size premium, or the average value premium will be positive.

Let's make the point concretely. Table 2 presents estimates of the likelihood of a negative equity premium (bills beat stocks) for periods of increasing length. The calculations use the observed average annual equity premium for 1963–2011 (5.95% per year) and the standard deviation of the year-by-year returns (17.85%) as the true expected value and volatility (thus the label $\text{Prob} < 0, E(RM - RF) = 5.95\%$). The calculations assume that annual observations on the equity premium are normally distributed; in fact, the distribution is fat-tailed (more outliers than predicted if returns are normal) and right-skewed (more extreme positive than negative returns). Statistical theory (the central limit theorem) tells us, however, that the distribution of the average premium converges toward the normal as the period covered by the average increases.

The mean of the annual equity premium for 1963–2011 is large, 5.95% per year. This is what attracts investors to stocks. The standard deviation of the premium is also large, 17.85%, or three times the mean. The year-by-year values of the premium are volatile. Again, investment in stocks is risky. Table 2 shows that if the true expected value of the annual equity premium is 5.95% and the standard deviation is 17.85%, a normal distribution predicts that the annual equity premium is negative—stocks underperform T-bills—in 37% of the years in 1963–2011. In fact, 33% (16 of 49 in Appendix Table A1) are negative.

The probability that the average stock return is below the average bill return falls if we lengthen the investment period. We are using simple average (not compound) returns, so the standard deviation of

the average annual premium for a period is the standard deviation of year-by-year premiums divided by the square root of the number of years in the period. Thus, if the period covered by the average premium is greater than a year, the standard deviation of the average is less than the standard deviation of year-by-year annual returns. Table 2 shows that the standard deviation of the average equity premium drops from 17.85% for a one-year period to 7.98% for five years, 5.65% for ten years, 3.57% for 25 years, and 2.53% for 50 years. Note the effect of the square root rule. The standard deviation of the average return falls at a slower and slower rate as the holding period is increased.

Because our assumed expected annual premium, 5.95%, does not change when the investment horizon changes, the decline in the standard deviation of the average reduces the likelihood of realizing a negative average premium as the horizon grows. Table 2 shows that the probability of negative average premium falls from 37.0% for a one-year period to 22.8% for 5 years, 14.6% for ten years, 4.8% for 25 years, and 0.9% for 50 years. In other words, looking forward, there is almost a one-in-four chance that the average premium for a five-year period is negative, that is, bills beat stocks. At 25 years—a moderate investment lifetime—there is still almost a one-in-twenty chance that the realized average equity premium is negative. It takes a long investment lifetime, 50 years, to reduce the probability of a negative realized average equity premium to 0.9%.

To emphasize, even if we know for sure that the true expected value of the equity premium is a healthy 5.95% per year, there is still a 0.9% chance that an investor who holds the market portfolio of stocks for the next 50 years ends up with a lower average annual return than an investor who rolls over one-month T-bills. Indeed, no matter how long the investment period, one can never be perfectly certain that the realized average premium will be positive. Such is the nature of risk and return!

There is another way to interpret Table 2. The discussion above treats the sample average equity premium for 1963–2011 as the true expected premium. But we don't know the value of the true expected premium, and Table 2 summarizes our uncertainty about it. The table says that if we have 50 years of data in which the average premium is 5.95% per year and the standard deviation of the year-by-year premiums is 17.85%, then our best estimate of the true expected premium is 5.95%, but it is subject to substantial

uncertainty, summarized by the estimate of the standard deviation of the 50-year average premium, 2.53%. Roughly speaking, with this 50 years of data, the probability distribution of the true expected premium is normal with mean 5.95% and standard deviation 2.53%. Thus, the true expected premium may be much bigger or much smaller than 5.95%, and there is a 0.9% chance that the true expected premium is negative. In general, even if the true expected premium does not change through time, we never have enough data to infer its value with perfect certainty. This is another way to think about risk in equity investing: We don't know the true expected values of risk premiums.

Unfortunately, uncertainty about true expected risk premiums makes us even less confident that investors will earn positive premiums in any given future period. In essence, we have to add uncertainty about expected premiums to the estimates of the uncertainty about investment outcomes obtained when we treat the sample mean premiums as true expected values.

To be precise (and you can skip this paragraph and the next if you are willing to take our word for it), the standard deviation of a future investment outcome over some fixed horizon is the square root of the sum of our estimate of the variance of the expected component and the variance of the unexpected component,

$$s(\text{Investment Outcome}) = [\text{var}(\text{Expected Return}) + \text{var}(\text{Unexpected Return})]^{1/2}.$$

We take the variance of the expected return in this equation to be the variance of the estimate of the 50-year average premium, that is, the square of $s(\text{Mean})$ in Table 2 for 50 Years (the last column). In other words, we take the uncertainty about the expected premium to be like the volatility of estimates of the average premium for a 50-year sample period (which is relatively low). The variance of the unexpected return for a given investment period (1 to 50 Years in Table 2) is then the square of $s(\text{Mean})$ for the given investment period. The overall uncertainty, expressed in the equation above, is the square root of the sum of the two variances. Note that the variance of the expected return is the same for all investment horizons, but the variance of the unexpected return is lower for longer investment periods.

Table 2 reports the likelihood that each of the three premiums will be negative over various investment horizons when we recognize that uncertainty about the true expected premiums adds to our

uncertainty about investment outcomes (labeled Prob < 0, E(RM – RF) uncertain). Uncertainty about true expected premiums has little effect on the uncertainty about the premiums for a one-year holding period. For example, recognizing our uncertainty about the expected value increases the probability of a negative realized equity premium in any particular year from 37.0% to 37.1%. Intuitively, for a one-year investment horizon, uncertainty about the expected premium is small relative to the high volatility of year-by-year returns. Uncertainty about the expected premium does not change with the investment horizon, but uncertainty about the unexpected component of the investment outcome goes down. As a result, uncertainty about the expected value has a bigger relative impact at longer horizons, increasing the probability of a negative realized average equity premium from 4.8% to 8.7% for a 25 year horizon and from 0.9% to 4.8% for 50 years. Thus, largely because of the uncertainty about the expected premium, even over a long 50-year investment lifetime, an investor has a one-in-twenty chance of a negative realized average equity premium.

The stories for the size and value premiums, SMB and HML, are similar to that for the equity premium. During 1963–2011 the size premium averages 3.69% per year, but confirming the impression drawn from Figure 2, the volatility of year-by-year values of the premium is high, with a standard deviation of 14.16% per year. As a result, Table 2 shows that if the true expected premium is 3.69% per year, the probability that the realized premium will turn out to be negative falls from about 39.7% for a one-year period to 28% for five years, 20.5% for ten years, and almost 10% for 25 years. In other words, even if we know for certain that small stocks are expected to beat big stocks by 3.69% per year, there is still a one in ten chance that big stocks produce a larger average return in a future 25-year period. Even for 50-year periods, there is a 3.3% probability that the average size premium turns out to be negative.

Alternatively, if we acknowledge that we don't know the true expected size premium and 50 years of data produce an average SMB return of 3.69% per year with a standard deviation of 14.16% per year, then the distribution of the expected premium is centered at 3.69% with a standard deviation of 2%, so again there is substantial uncertainty about the value of the true expected annual premium. It could be much bigger than 3.69% per year, but it could be much smaller, and there is a 3.3% chance that it is

negative. Acknowledging this uncertainty increases the likelihood of a negative average size premium to about 10% for investors with a long, 50-year investment horizon.

Prospects are a bit rosier for the value premium. The average HML return for 1963–2011 is 5.29% per year, which is similar to the average equity premium, 5.95%. Though large at 13.94%, the standard deviation of the HML return is less than that of the equity premium, 17.85%. As a result, if the true expected value premium is 5.29%, the probability that a negative average value premium will be observed in a period of given length is lower than for the equity premium. Still, the probability of a negative average value premium for a five-year period is almost 20%, falling to 11.5% at ten years, and 2.9% at 25 years (Table 2).

Alternatively, if we acknowledge that we don't know the true expected premium and 50 years of data produce an average HML return of 5.29% per year with a standard deviation of 13.94% per year, then the distribution of the expected premium is centered at 5.29% with a standard deviation of 1.97% (close to that of the size premium), so again there is substantial uncertainty about the value of the true expected annual premium. It could be much bigger than 5.29% per year, but it could be much smaller, and there is a 0.4% chance that it is negative. Combining both sources of uncertainty, the probability of a negative value premium is 6.1% over the next 25 years and 2.9% over the next 50 years.

In sum, investing in stocks is risky. Even over a long investment lifetime, there is no guarantee that stocks will beat bills, or that small stocks will beat big stocks, or that value stocks will beat growth stocks. Moreover, our analysis is based on highly diversified portfolios that are probably rather free of diversifiable volatility. For less diversified portfolios, uncertainty about outcomes is higher. This is true for actively managed stock portfolios, which are undiversified almost by definition. And it's true in spades for riskier investments like private equity and hedge funds. Caveat emptor!

Figure 1: Annual Market Return in Excess of the T-Bill Rate

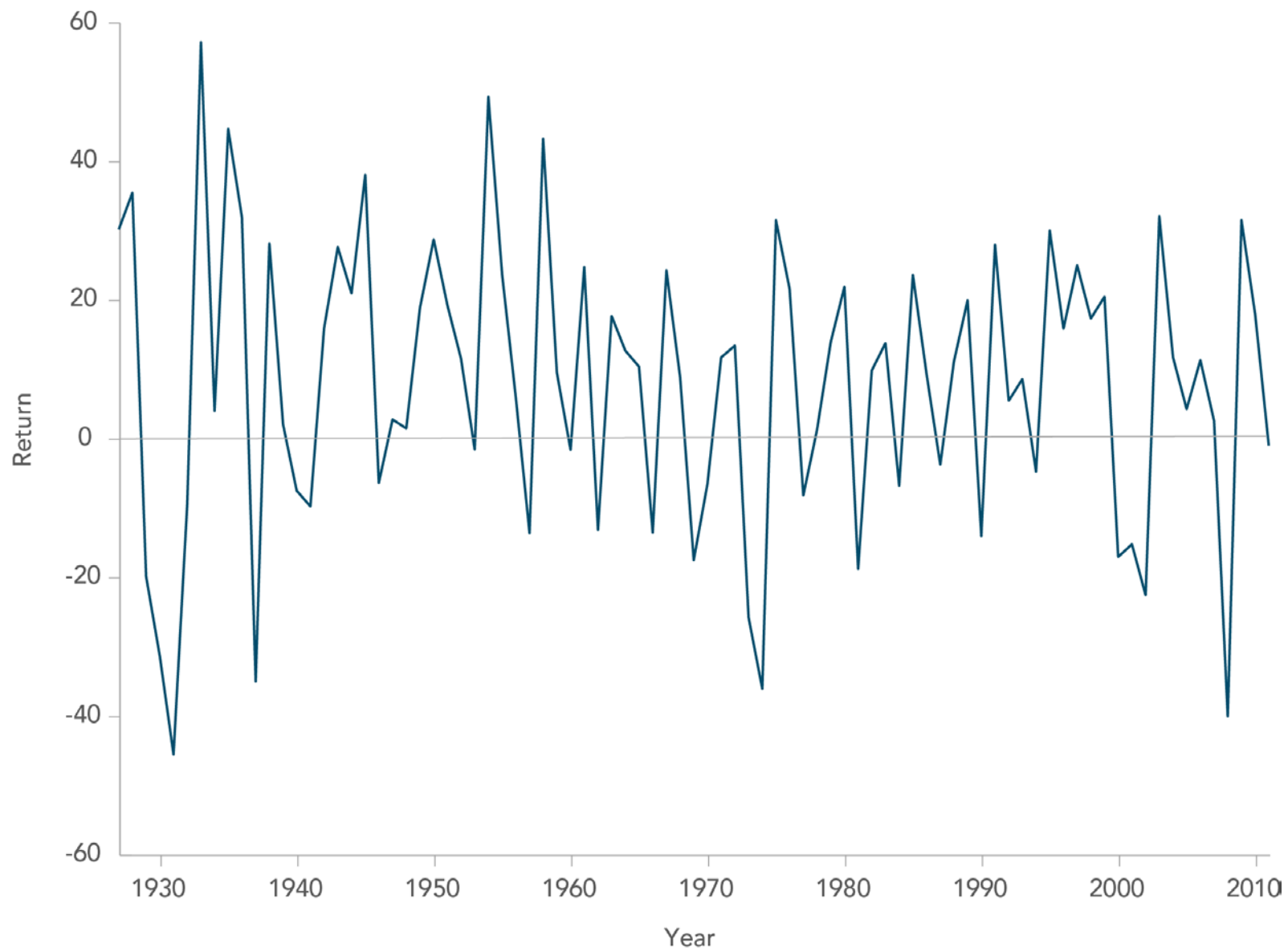


Figure 2: Annual RM – RF and SMB Returns

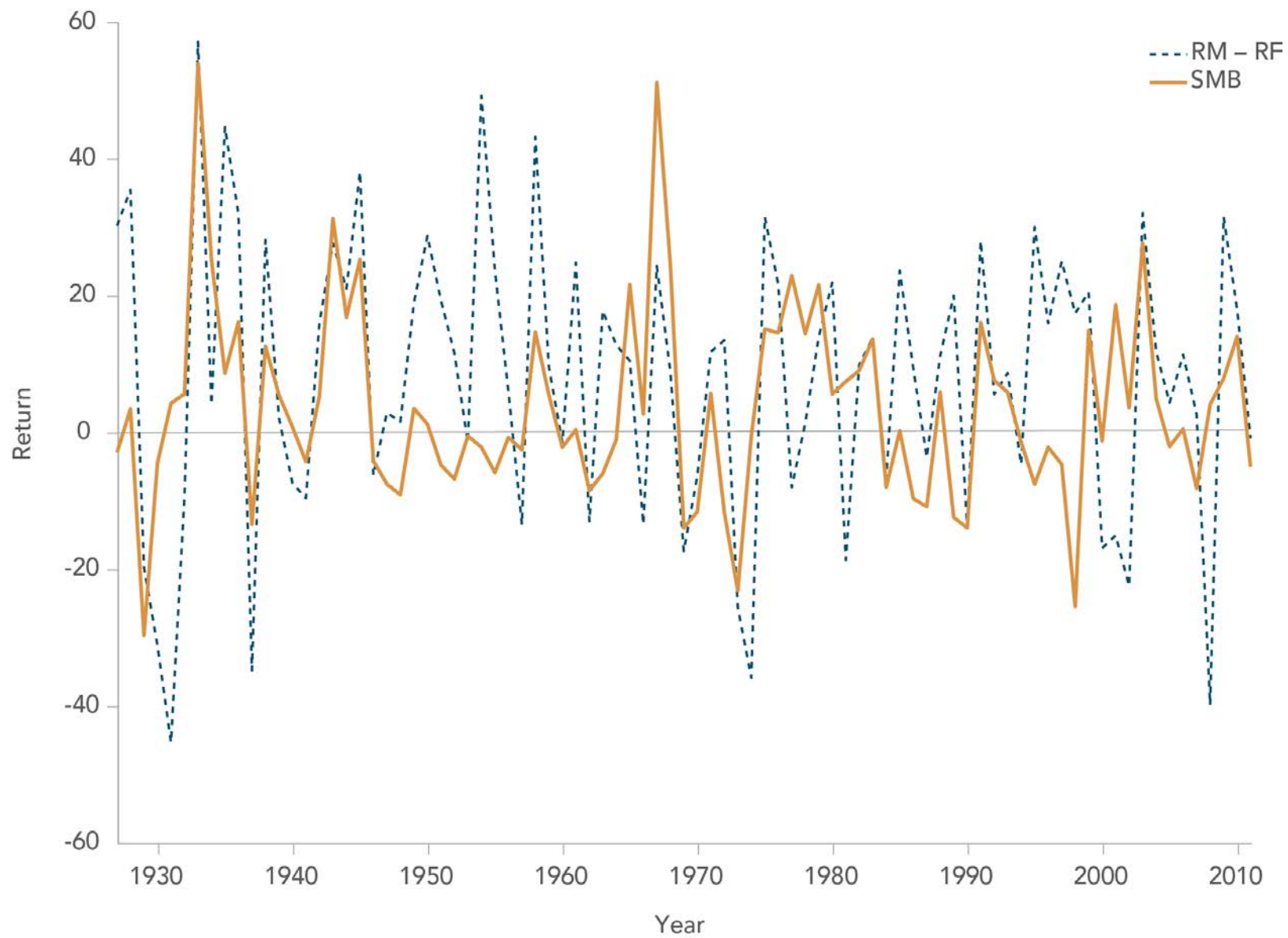


Figure 3: Annual RM – RF and HML Returns

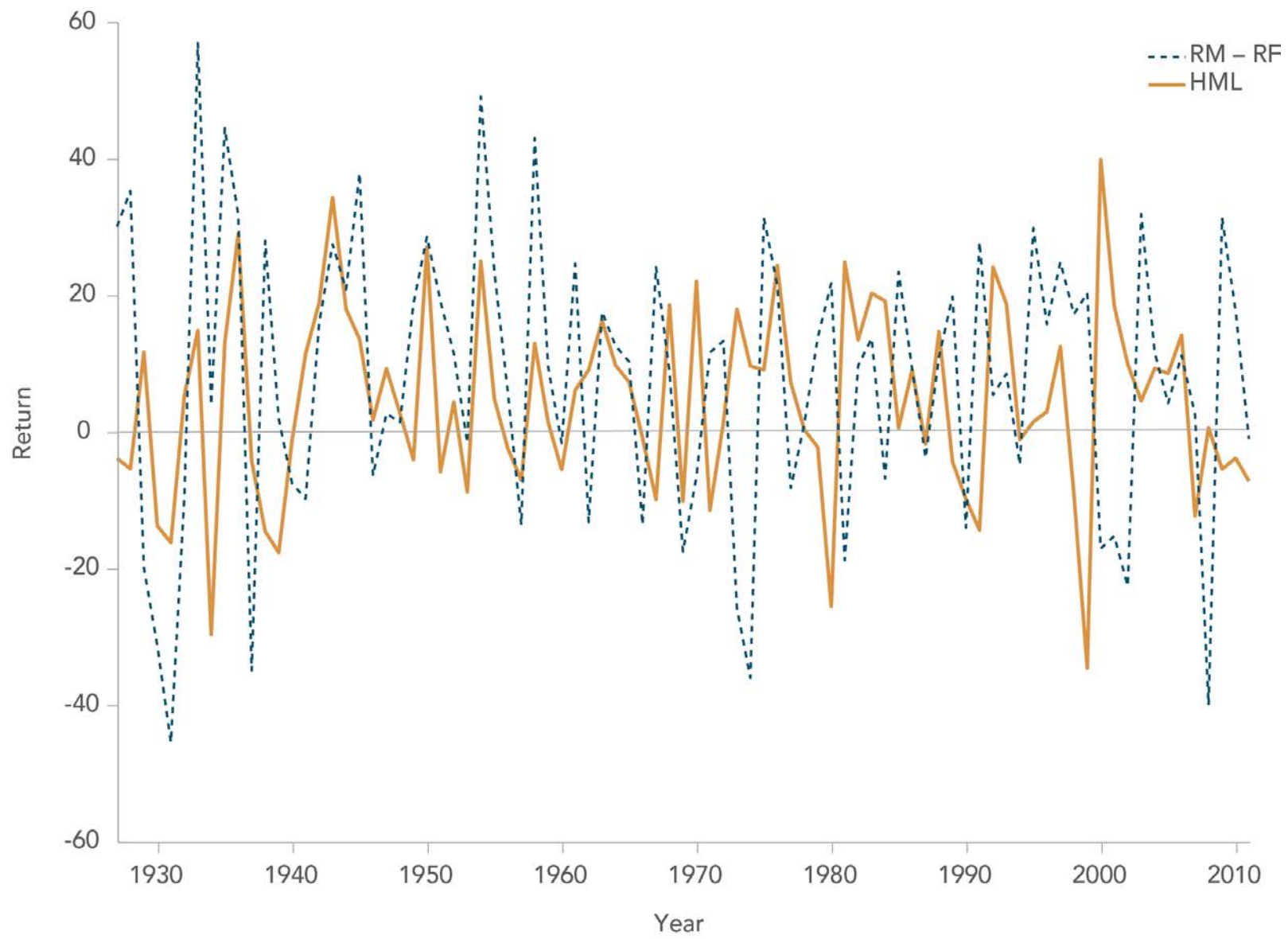


Table 1: Summary statistics for annual returns for 1963–2011, 49 years

The table shows averages (Mean) and standard deviations (Std) of annual values of the equity premium, $RM - RF$, the size premium, SMB, and the value premium, HML. The equity premium is the difference between the market return, RM , and the T-bill return, RF . The size premium is the average of the returns on the three portfolios of small stocks (SG, SN, and SV) minus the average of the returns on the three portfolios of big stocks (BG, BN, and BV). The value premium is the average of the returns on the two value portfolios (SV and BV) minus the average of the returns on the two growth portfolios (SG and BG).

	RM – RF	SMB	HML					
Mean	5.95	3.69	5.29					
Std	17.85	14.16	13.94					

	RM	SG	SN	SV	BG	BN	BV	RF
Mean	11.27	11.58	16.87	19.23	10.96	11.76	13.89	5.33
Std	17.71	28.14	22.91	24.22	18.62	16.95	18.61	2.99

Table 2: Probability that an average premium is negative in periods of different length

Years is the number of years in a period, and $\sqrt{\text{Years}}$ is the square root of Years. $s(\text{Mean})$ is the standard deviation of the average premium, which is the standard deviation of year-by-year values of the premium multiplied by $1/\sqrt{\text{Years}}$. $\text{Prob} < 0$ is the probability that the realized average premium in a period of length Years is negative. We compute two probabilities for each premium. The first assumes the average premium for 1963–2011 is the true expected premium and the second recognizes our uncertainty about the true expected premium. The means of the year-by-year annual premiums for 1963–2011 are 5.95% (RM – RF), 3.69% (SMB), and 5.29% (HML) per year, and the standard deviations are 17.85% (RM – RF), 14.16% (SMB), and 13.94% (HML). $\text{Mean}/s(\text{Mean})$ tells us how far the mean is from zero, measured in units of the standard deviation of the mean. For readers interested in the details, when we assume we know the true expected premium, the probability that the realized average premium is negative in a period of length (Years) is estimated as the probability to the left of $\text{Mean}/s(\text{Mean})$ from a unit normal distribution. When we acknowledge our uncertainty about the true expected premiums, our uncertainty about future realized returns is the square root of the sum of the variance of the average return if we knew the true expected premium and the variance of our estimate of the premium. Since we assume we use 50 years to estimate the expected value, the variance of our estimate is the square of $s(\text{Mean})$ at 50 years.

Years	1	5	10	15	20	25	50
$1/\sqrt{\text{Years}}$	1.00	0.45	0.32	0.26	0.22	0.20	0.14
Equity Premium, RM – RF							
$s(\text{Mean})$	17.85	7.98	5.65	4.61	3.99	3.57	2.53
$\text{Mean}/s(\text{Mean})$	0.33	0.74	1.05	1.29	1.49	1.67	2.35
$\text{Prob} < 0, E(\text{RM-RF}) = 5.95\%$	37.0%	22.8%	14.6%	9.9%	6.8%	4.8%	0.9%
$\text{Prob} < 0, E(\text{RM-RF})$ uncertain	37.1%	23.9%	16.8%	12.9%	10.4%	8.7%	4.8%
Size Premium, SMB							
$s(\text{Mean})$	14.16	6.33	4.48	3.66	3.17	2.83	2.00
$\text{Mean}/s(\text{Mean})$	0.26	0.58	0.82	1.01	1.16	1.30	1.84
$\text{Prob} < 0, E(\text{SMB}) = 3.69\%$	39.7%	28.0%	20.5%	15.7%	12.2%	9.7%	3.3%
$\text{Prob} < 0, E(\text{SMB})$ uncertain	39.8%	28.9%	22.6%	18.8%	16.3%	14.4%	9.7%
Value Premium, HML							
$s(\text{Mean})$	13.94	6.24	4.41	3.60	3.12	2.79	1.97
$\text{Mean}/s(\text{Mean})$	0.38	0.85	1.20	1.47	1.70	1.90	2.68
$\text{Prob} < 0, E(\text{HML}) = 5.29\%$	35.2%	19.8%	11.5%	7.1%	4.5%	2.9%	0.4%
$\text{Prob} < 0, E(\text{HML})$ uncertain	35.4%	20.9%	13.7%	9.9%	7.6%	6.1%	2.9%

Appendix Table A1: Annual Returns, 1963–2011: 49 Years

	RM	SG	SN	SV	BG	BN	BV	RM-RF	SMB	HML	RF
1963	20.89	7.54	16.52	29.68	21.95	17.08	32.82	17.78	-6.04	16.51	3.11
1964	16.30	8.64	17.73	23.87	14.41	19.89	19.15	12.78	-1.07	9.98	3.53
1965	14.39	34.91	33.44	42.06	14.60	8.38	22.51	10.47	21.63	7.53	3.92
1966	-8.69	-5.94	-6.10	-7.07	-10.78	-5.47	-10.93	-13.44	2.69	-0.64	4.75
1967	28.56	90.59	71.83	68.13	28.91	15.69	32.10	24.35	51.28	-9.63	4.21
1968	14.16	32.42	40.65	46.82	3.84	15.31	27.02	8.94	24.57	18.79	5.22
1969	-10.85	-24.65	-22.76	-25.39	2.56	-16.79	-16.45	-17.42	-14.04	-9.88	6.57
1970	0.06	-21.49	-7.65	6.44	-6.09	7.87	10.49	-6.45	-11.66	22.25	6.52
1971	16.19	26.37	21.48	14.46	24.83	5.92	14.42	11.80	5.72	-11.16	4.39
1972	17.33	0.77	7.50	7.33	21.39	11.18	18.12	13.50	-11.70	1.65	3.84
1973	-18.75	-45.24	-32.72	-27.17	-22.31	-9.07	-4.04	-25.68	-23.24	18.17	6.93
1974	-27.94	-32.41	-25.85	-18.61	-29.21	-22.82	-23.22	-35.95	-0.54	9.89	8.01
1975	37.35	60.82	58.02	58.12	34.23	41.86	55.59	31.55	15.10	9.33	5.80
1976	26.77	38.43	47.43	60.65	17.61	40.58	44.48	21.68	14.61	24.55	5.08
1977	-2.97	18.81	18.44	23.26	-9.14	-0.61	1.35	-8.10	22.97	7.47	5.13
1978	8.53	17.77	21.06	22.27	7.07	6.93	3.76	1.34	14.45	0.59	7.19
1979	24.39	49.61	38.00	39.53	16.41	23.43	22.50	14.01	21.60	-2.00	10.38
1980	33.25	53.19	30.58	22.05	35.77	36.82	16.41	21.99	5.61	-25.25	11.26
1981	-3.99	-10.65	14.26	17.68	-7.70	-7.42	14.09	-18.71	7.44	25.06	14.72
1982	20.43	19.47	33.41	41.17	21.68	17.79	27.41	9.90	9.06	13.72	10.53
1983	22.66	19.78	40.61	47.94	14.55	25.40	27.43	13.87	13.65	20.52	8.80
1984	3.17	-13.83	2.55	8.32	-0.52	5.68	16.10	-6.67	-8.07	19.38	9.84
1985	31.41	29.76	35.22	32.41	32.49	32.60	31.56	23.69	0.25	0.86	7.72
1986	15.55	2.47	10.06	14.45	14.73	20.11	21.32	9.40	-9.72	9.28	6.16
1987	1.81	-13.33	-4.35	-6.62	7.57	3.02	-2.14	-3.66	-10.91	-1.50	5.47
1988	17.56	14.61	28.61	31.06	12.67	17.87	26.05	11.20	5.89	14.92	6.36
1989	28.42	19.44	17.91	17.05	35.93	25.73	30.12	20.04	-12.46	-4.10	8.38
1990	-6.09	-18.35	-17.91	-23.63	0.89	-5.74	-12.97	-13.93	-14.02	-9.57	7.84
1991	33.63	53.30	46.88	40.80	43.11	22.18	27.46	28.03	16.07	-14.07	5.60
1992	9.06	4.43	22.49	35.59	6.27	9.65	23.78	5.56	7.60	24.33	3.50
1993	11.58	10.59	20.17	26.95	0.85	16.89	22.40	8.68	5.86	18.95	2.90
1994	-0.75	-6.56	0.19	0.01	2.60	1.09	-5.72	-4.66	-1.44	-0.88	3.91
1995	35.67	29.10	28.03	33.34	37.49	39.11	36.74	30.07	-7.62	1.74	5.60
1996	21.15	9.61	22.67	24.18	22.90	25.38	14.72	15.95	-2.18	3.19	5.20
1997	30.33	10.05	32.22	38.93	30.70	37.30	27.24	25.08	-4.68	12.71	5.25
1998	22.28	-1.67	-5.68	-0.79	39.45	7.99	21.06	17.42	-25.54	-8.76	4.85
1999	25.23	47.49	21.97	7.29	27.05	6.27	-1.24	20.54	14.89	-34.24	4.69
2000	-11.08	-23.15	19.57	23.24	-13.45	16.67	20.23	-16.96	-1.27	40.04	5.88
2001	-11.27	0.03	16.98	22.93	-14.67	-1.25	-0.22	-15.13	18.70	18.68	3.86
2002	-20.85	-31.95	-11.44	-9.08	-22.58	-15.55	-25.09	-22.48	3.59	10.18	1.63
2003	33.14	54.81	50.09	64.61	27.93	30.75	27.74	32.12	27.69	4.80	1.02
2004	12.99	15.03	20.60	21.56	7.53	14.79	20.06	11.80	4.94	9.53	1.19
2005	7.33	-0.22	8.52	9.45	4.07	7.96	12.00	4.35	-2.09	8.80	2.98
2006	16.21	8.88	22.25	23.37	9.49	19.77	23.83	11.40	0.47	14.41	4.81
2007	7.32	5.11	-2.53	-10.81	11.42	1.76	3.26	2.65	-8.22	-12.04	4.67
2008	-38.30	-38.90	-31.25	-31.79	-33.67	-41.42	-39.09	-39.94	4.08	0.85	1.64
2009	31.62	34.87	31.80	34.07	32.47	21.66	22.87	31.57	7.91	-5.20	0.05
2010	18.05	30.33	29.70	28.65	15.75	20.58	10.28	17.97	14.03	-3.58	0.08
2011	-0.90	-3.21	-4.74	-6.52	3.91	3.39	-6.64	-0.91	-5.04	-6.93	0.01